Pricing Short-Circuit Current via a Primal-Dual Formulation for Preserving Integrality Constraints

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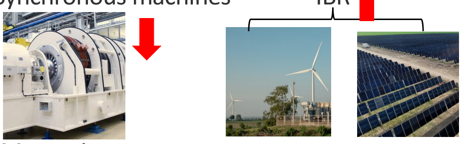
Background & Motivation

Synchronous machines — IBR



Magnetic energy — Electronic converters

5–8 p.u. SCC
A strong electrical signal that can be detected by grid protective relays, ensuring they clear faults reliably.

1–3 p.u. SCC, even lower
A weaker fault current due to their electronic limits. This weak signal may be insufficient to trigger the protection, leaving faults uncleared.

Background:

High penetration of inverter-based resources (IBR) displaces synchronous generators (SGs). IBR deliver limited short-circuit current (SCC), reducing system-wide SCC and endangering relay protection. SCC has evolved into an essential ancillary service. Unit commitment (UC) binary variables cause non-convexity and hinder accurate SCC pricing.

Motivation:

- Dispatchable pricing relaxes integrality, distorting SCC modelling, underpricing and requiring uplift payments. Restricted pricing bundles SCC with other start-up-linked services, fails independent SCC valuation and excludes remuneration for synchronous condensers without commitment variables.
- Derive explicit nodal SCC shadow prices, which are applicable to synchronous condensers, while preserving UC integrality and accurate SCC formulation.
- Guarantee full cost recovery for generators via market prices and eliminate uplifts.

Methodologies & Models

Short-Circuit Current Constraints:

For a power system incorporating SGs and IBR, the SCC at bus b due to their current injections can be expressed as:

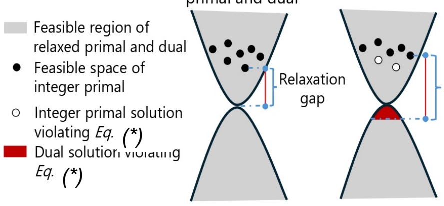
$$I_{b_{SC}} = \frac{\sum_{g\in\mathcal{G}} Z_{b\Psi(g)} \mathrm{I}_g u_g + \sum_{c\in\mathcal{C}} Z_{b\Phi(c)} \mathrm{I}_c \alpha_c}{Z_{bb}}$$

Offline training to approximate actual SCC

$$I_{b_L} = \sum_g k_{bg} u_g + \sum_c k_{bc}\alpha_c + \sum_m k_{bm}\eta_m \geq \mathrm{I}_{b_{lim}}$$

$$\eta_m = u_{g_1}\cdot u_{g_2}, \quad \text{s.t. } \{g_1, g_2\} = m$$

$$m \in \mathcal{M} = \{g_1, g_2 \mid \forall g_1, g_2 \in \mathcal{G}\}$$



$$\min_{V} \ \mathbf{Obj}_{primal} - \mathbf{Obj}_{dual}$$

where:

$$V = \left\{ V_{primal}, V_{dual} \right\}$$

subject to:

Primal constraints

Dual constraints

Non-negative profit constraint

$$\underbrace{\lambda^{\mathrm{E}} P_g}_{\text{Energy revenue}} + \underbrace{\sum_b \lambda_b^{\mathrm{SCC}} \mathrm{k}_{bg} u_g + \sum_b \sum_{\{m|g\in m\}} \lambda_b^{\mathrm{SCC}} \mathrm{k}_{bm}\eta_m}_{\text{SCC revenue}} - \underbrace{(\mathrm{c}_g^{\mathrm{nl}} u_g + \mathrm{c}_g^{\mathrm{m}} P_g + C_g^{\mathrm{st}})}_{\text{Operating cost}} \geq 0, \quad \forall g$$

Primal

$$\min_{V_{\mathrm{P}}} \sum_g (\mathrm{c}_g^{\mathrm{nl}} u_g + \mathrm{c}_g^{\mathrm{m}} P_g + C_g^{\mathrm{st}})$$

subject to:

$$\sum_g P_g + \sum_c P_c = \mathrm{P}^{\mathrm{D}} : (\lambda^{\mathrm{E}})$$

$$u_g \mathrm{P}_g^{\min} \leq P_g \leq u_g \mathrm{P}_g^{\max} : (\mu_g^{\min}, \mu_g^{\max}), \ \forall g$$

$$C_g^{\mathrm{st}} \geq (u_g - u_{g,0})\mathrm{c}_g^{\mathrm{st}} : (\sigma_g^{\mathrm{st}}), \ \forall g$$

$$0 \leq P_c \leq \alpha_c \mathrm{P}_c^{\max} : (\zeta_c^{\min}, \zeta_c^{\max}), \ \forall c$$

$$u_g \in \{0,1\}, \ \forall g$$

SCC constraint : $(\lambda_b^{\mathrm{SCC}}), \ \forall b$

McCormick envelopes for linearizing $\eta_m, \ \forall m$

Dual

$$\max_{V_{\mathrm{D}}} \ \mathrm{P}^{\mathrm{D}}\lambda^{\mathrm{E}} + \sum_b (\mathrm{I}_{b_{\lim}} - \sum_c \mathrm{k}_{bc}\alpha_c)\lambda_b^{\mathrm{SCC}} - \sum_c \alpha_c \mathrm{P}_c^{\max}\zeta_c^{\max} - \sum_g \psi_g^{\max} - \sum_m \gamma_{m,1}^{\min} - \sum_a u_{g,0}\mathrm{c}_g^{\mathrm{st}}\sigma_g^{\mathrm{st}}$$

subject to:

$$\mathrm{c}_g^{\mathrm{nl}} - \sum_b \mathrm{k}_{bg}\lambda_b^{\mathrm{SCC}} - \mathrm{P}_g^{\max}\mu_g^{\max} + \mathrm{P}_g^{\min}\mu_g^{\min} + \mathrm{c}_g^{\mathrm{st}}\sigma_g^{\mathrm{st}} + h_g(\gamma_{m,1}^{\max}, \gamma_{m,2}^{\max}, \gamma_{m,1}^{\min}) + \psi_g^{\max} \geq 0, \ \forall g$$

$$\mathrm{c}_g^{\mathrm{m}} - \lambda^{\mathrm{E}} + \mu_g^{\max} - \mu_g^{\min} \geq 0, \ \forall g$$

$$1 - \sigma_g^{\mathrm{st}} \geq 0, \ \forall g$$

$$-\lambda^{\mathrm{E}} + \zeta_c^{\max} \geq 0, \ \forall c$$

$$\gamma_{m,1}^{\max} + \gamma_{m,2}^{\max} - \gamma_{m,1}^{\min} - \sum \mathrm{k}_{bm}\lambda_b^{\mathrm{SCC}} \geq 0, \ \forall m$$

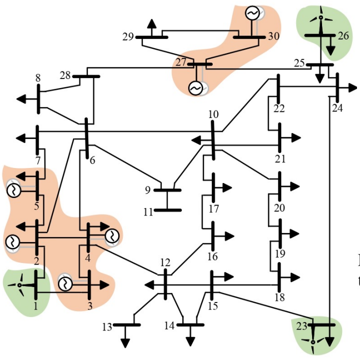
$$\{V_{\mathrm{D}} | V_{\mathrm{D}} \neq \lambda^{\mathrm{E}}\} \in \mathbb{R}_+, \ \forall b, g, c, m$$

Primal-Dual Formulation:

A formulation that tries to minimize the duality gap, considering non-negative profit for each unit:

- **Primal of SCC-Constrained UC** (Non-convexity from binary variables) → *Relax binary variables* → **Dual of Relaxed SCC-Constrained UC**
- **Primal of SCC-Constrained UC** and **Dual of Relaxed SCC-Constrained UC** → **Primal-Dual Formulation** (Discretize relaxed binary variables)
- **Non-Negative Profit Constraint:** *Eq.* (*) → **Primal-Dual Formulation**

Results & Conclusion



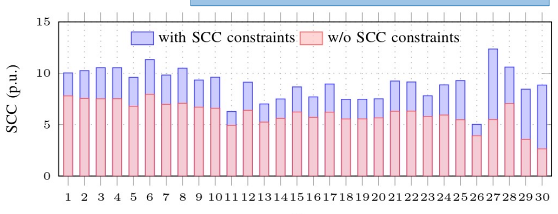


Fig. 8. Minimum SCC level at each bus with/without SCC constraints over the market horizon. The SCC threshold $\mathrm{I}_{b_{lim}} = 5$ p.u..

SCC levels can be secured by SGs

Price signals: Dispatchable pricing creates redundant SCC prices and undervalues critical buses. Primal-dual (P-D) pricing produces precise signals only at SCC-weak buses.

Market interaction: P-D pricing captures the complementary link between energy and SCC services, while dispatchable pricing disconnects the two markets.

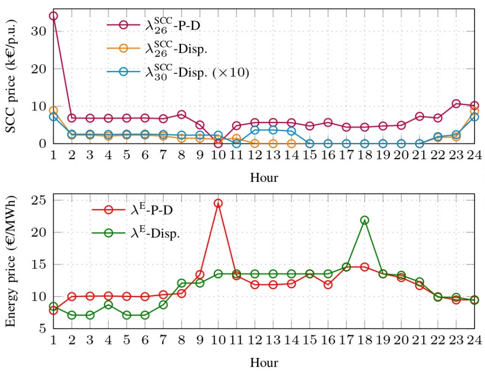


Fig. 9. Price profiles of SCC (upper) and energy (lower) over the complete market horizon. The SCC price for other buses is zero in each case.

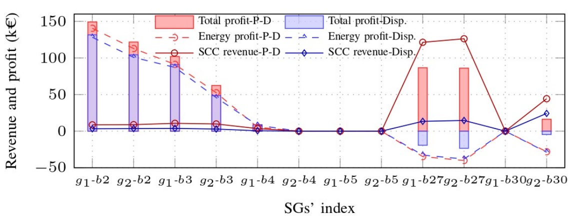


Fig. 10. Profitability of each SG under P-D and dispatchable pricing methods. Total profit is equal to the sum of energy profit and SCC revenue, in which the energy profit is energy revenue minus operating cost.

Generator revenue: Core SCC units suffer losses under dispatchable pricing. P-D pricing fairly compensates all units without uplifts.

Overall advantages: P-D pricing balances grid security and economic benefits, ideal for SCC pricing in high-renewable power systems.

arXiv GitHub

- https://arxiv.org/abs/2510.05293
- https://github.com/pwang30/Analyzing_the_Impact_of_DR_on_SCC_Level

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